1. Functional derivatives: For a three dimensional elastic medium, the potential energy is

$$V = \frac{\tau}{2} \int d^3x (\nabla\phi)^2,$$

and the kinetic energy is

$$T = \frac{\rho}{2} \int d^3x \left(\frac{\partial\phi}{\partial t}\right)^2.$$

Use the variational principle to show that ϕ obeys the wave equation

$$\nabla^2\phi = \frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2}$$

where v is the wave velocity, ρ the density of the medium and τ the tension.

2. Functional derivatives: Show that if

$$Z[J] = \exp\left[-\frac{1}{2}\int d^4x\, d^4y\, J(x)\,\Delta(x-y)\,J(y)\right],$$

where $\Delta(x) = \Delta(-x)$, then

$$\frac{\delta Z[J]}{\delta J(z_0)} = -Z[J] \int d^4y\, \Delta(z_0 - y)\, J(y)$$

3. In class we have evaluated the asymptotic forms of the d-dimensional Fourier transform to show that the correlation function has the Ornsetin-Zernike form. Now do this in three dimensions by carefully evaluating the contour integral! The Fourier transform

$$G(\vec{r}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d^3k\; e^{i\vec{k}\cdot\vec{r}} \tilde{G}(\vec{k}),$$

and

$$\tilde{G}(\vec{k}) = \frac{1}{k^2 + \xi^{-2}}.$$

What is $G(\vec{r})$ when $\xi \to \infty$?